Stability Analysis using Seepage Forces

INTRODUCTION

In the early days of my engineering career Stability Analyses were computed by hand calculation and were tedious, but nevertheless were a requirement for even small dams. So, for the handful of potential sliding surfaces which could be examined a great deal of thinking went into deciding what/where might be the weaknesses in the design section. And that contemplation was in my opinion more important than whatever safety factor came out of the calculations. Of course all that changed with the advent of computers and search routines.

While I was an enthusiastic user of computers, writing several of my own programs for stability computations, it became apparent to me that something was missing. This was especially so when it came to "end of construction" and "upstream rapid drawdown" pore pressure conditions. More recently the relatively straightforward "steady state seepage" condition for the downstream slope became worrisome, particularly in the light of the experiences at Tarbela and Bennett dams. Perhaps, I thought, it is time to take a fresh look at the prevailing notions about how the soil-structure actually behaved in the presence of energized water; taking for granted what seems evident, that - where there is no water there is no problem.

What is presented here is a "start from scratch" examination of the forces at play within a soil slope affected by seepage water. I find forces more tangible than stresses or pressures. When it comes to the all-important quantification of pore water pressures, this scalar quantity seems rather ethereal compared with a force vector which defines both magnitude and direction of action. Vectors can be drawn and resolved as a force polygon; this graphical representation of forces ensures that all vectors are apparent. And once the polygon closes we gain some confidence that we are doing the right thing. In consequence, examination of forces, as opposed to pressures or stresses, is adopted as the guiding principle on which this approach is founded.

This, let's call it a technical essay, has been written/presented in an open/exposed and explicit style with the simple intention of making it as vulnerable as possible to refutation. If it so happens that I'm wrong then I'll learn something important, but if I'm right I'll learn nothing more at all.

COMPUTATIONAL PROCEEDURE/METHOD

In line with accepted/common practice the "method of slices" will be used here.

Figure 1 shows the slope geometry which will herein be examined under various soil-structure and hydraulic constraints. It is a 2:1 granular fill embankment, and as shown there, founded on a cohesive deposit. The differential head between the pond level and the tailwater results in seepage flow through the soils. The

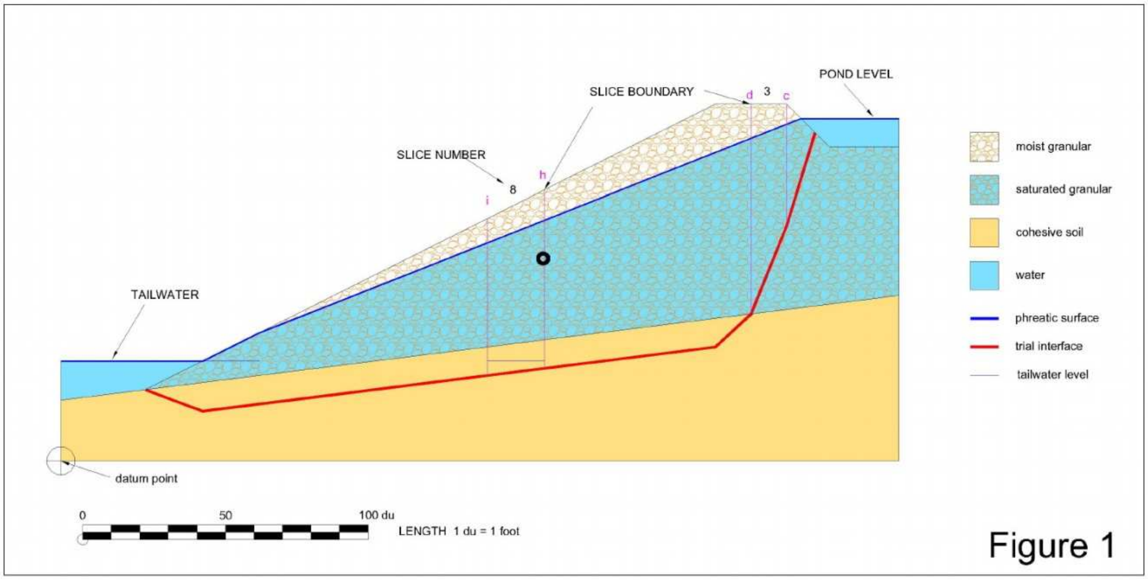


Figure 1

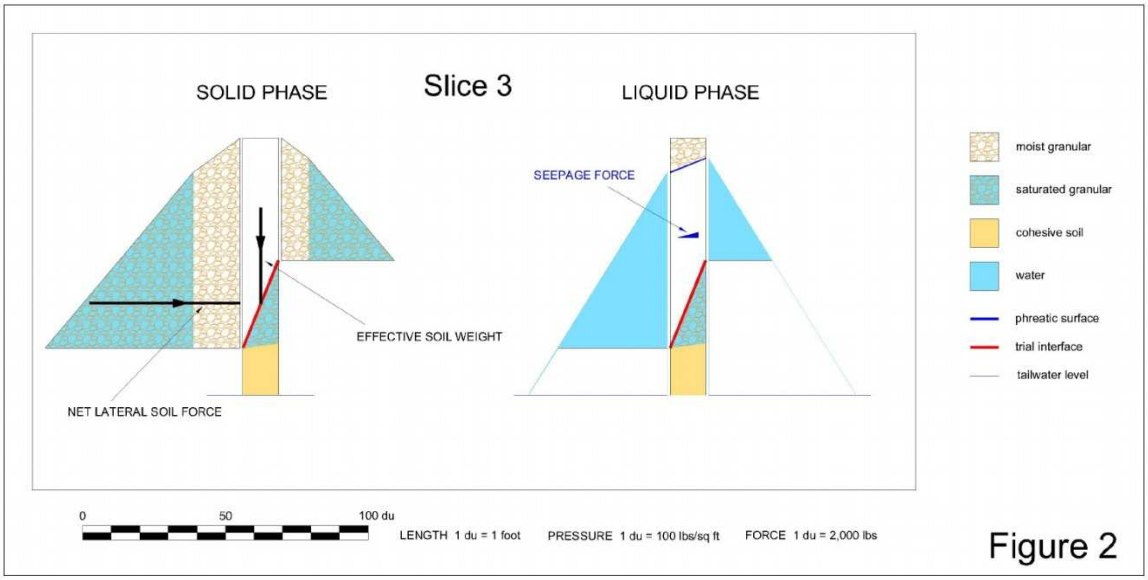
line defining the phreatic surface is somewhat arbitrary, and not based on flow net type analysis: This being so because in reality the position of the phreatic surface is entirely dependent on the ratio of horizontal to vertical permeability of the soils involved – a ratio which is almost never known.

The trial interface (failure zone) is also arbitrarily chosen. The black donut shows the centre of mass (effective) of the soils above the trial interface. The scale is referenced to "du" (drawing unit) to make it independent of modes of printing. Again, as a matter of personal choice the foot-pound system of units is used here. Following convention, volumes are numerically equal to areas on the understanding that thicknesses are uniformly 1 foot deep. As a final note before starting into the subject proper: All of the calculations for the results quoted here were made using a Microsoft Excel spreadsheet. A "live/active" version of that sheet is attached hereto in order that the reader may check/verify any/all the numerical values quoted here, as well as providing them the opportunity to explore the effects of changing the soil parameters at their whim, to see the resulting affect on slope stability computed by this method.

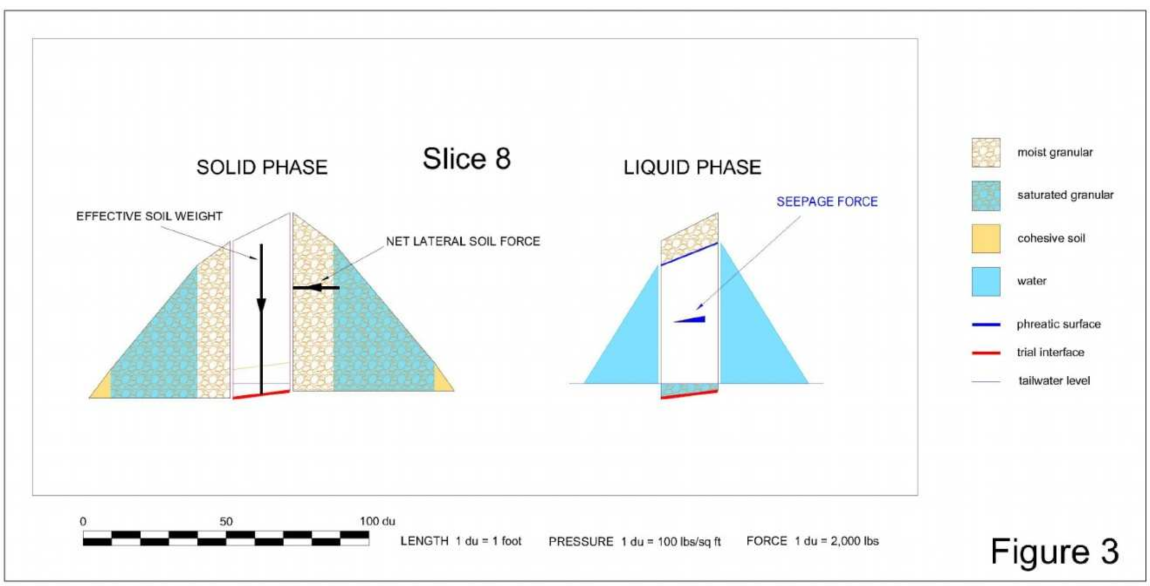
Figures 2 and 3 show the physical conditions associated with Slices 3 and 8, respectively. There are two sketches on each figure: The one on the left side depicts the solid soil-structure forces and the pressures from which these are derived; the one on the right deals with liquid phase - pore water. These two intimately related phases of the soil will be examined separately before combining their influence on stability. First, the solid phase.

<u>Solids of soil-structure</u>

Here the phreatic surface is used to discriminate between the moist soil above and the saturated soil below. The moisture content above is set at the



equivalent of 5% saturation, see Ref 1. Soils below the phreatic line are considered fully saturated, and consequently exert only their buoyant weight on the soil-structure beneath. This is consistent with standard usage as in the equation for shear strength, $s = c + (\sigma_n - u) \tan \Phi$ where the parenthetical term is equivalent to the sense used here.



The resultant vertical force acting on the trial interface is shown to scale, as is the resultant of the horizontal force brought about by lateral soil pressures on the two vertical sides of the slice. These latter forces are of no consequence to stability since over the length of the stability section these obviously balance out, and all that remains is the difference in hydraulic/hydrostatic force between the first and last slices. They are only drawn here to indicate the relative magnitudes of the forces involved, and also make it apparent that they have not been ignored.

Figure 4 shows the force polygons for Slices 3 and 8 as derived from Figures 2 and 3. The two force magnitudes of interest here are:

a. P, the component of the effective soil weight (W') parallel to the base of the slice, from which we find "H", the force which is consequent upon the inclination of the interface, and

b. S, the ultimate shear resistance to translation along the base.

These two pieces of information from each of the slices, together with the knowledge that inter-slice lateral forces cancel out, we have learned all that is relevant to stability from the solid phase geometry.

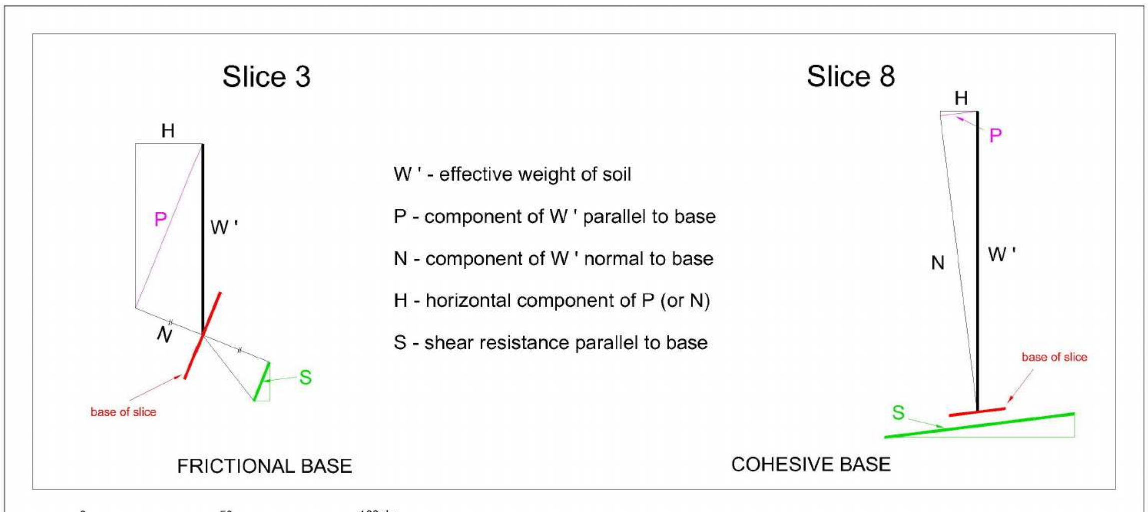


Figure 4

The methodology underlying this analytical approach is to treat the mass above the trial interface as a "free-body", that is to say, only boundary restraints to movement, and forces emanating from within the body itself need be considered in order to establish its stability with respect to translation along the interface.

Now that we have accounted for the soil phase, we must next consider the liquid phase. The fact that steady state seepage is occurring within the soil-structure, a persistent event which leads to energy being expended within the system, is a reality which must be taken into account.

Water within the soil-structure

To establish that steady state seepage is an important component of slope stability it is only necessary to appreciate what is shown in Figure 5.

This is a photograph taken through the glass wall of a water filled test tank. The water level is indicated by the hand-drawn arrow in the upper right hand corner. Clean sand was placed loosely within a latex membrane. A drainage layer of fine crushed rock (the lighter colour) underlay the sandfill. This drain was vented to the atmosphere by a vertical outlet. Then, after submergence within the tank, the membrane was gradually pulled up to expose the sandfill directly to the surrounding water. The dry (subaerial) angle of repose of this sand is 34°. Here it may be seen to stand with a vertical side slope underwater. The reason for this counterintuitive behaviour is the creation of an inward hydraulic gradient due to the differential radial water pressures, in other words, a phreatic surface inclined from the sand face to the inner drain.

Figure 5

Obviously, this real and powerful phenomenon ought to be made part of stability analyses computations.

SEEPAGE FORCES

The essential starting point in appreciation Seepage Forces [SF] is the realization that pore water pressures, when quiescent/hydrostatic, have no influence whatever on the buoyant soil-structure: It is only pore pressure gradients that affect the soil-structure, and thereby a slope's stability.

In Figure 6 we can see how the declining phreatic surface may be interpreted and translated into a Seepage Force [SF], such as that responsible for the vertical slopes in Figure 5. The diagram on the left shows the hydraulic conditions that prevail in the liquid phase, using Slice 8 of Figure 1 as a typical case in point. The mathematical proposition on the right hand side here explains that the inclination of the phreatic surface is really the maximum hydraulic gradient [*i*] prevailing within the saturated soils beneath. And knowing this, we can quantify the SF acting within this slice as:

$$SF = i \ \gamma_w \ V$$

where γ_w is the unit weight of water, and V is the area (volume of unit thickness) subject to flow.

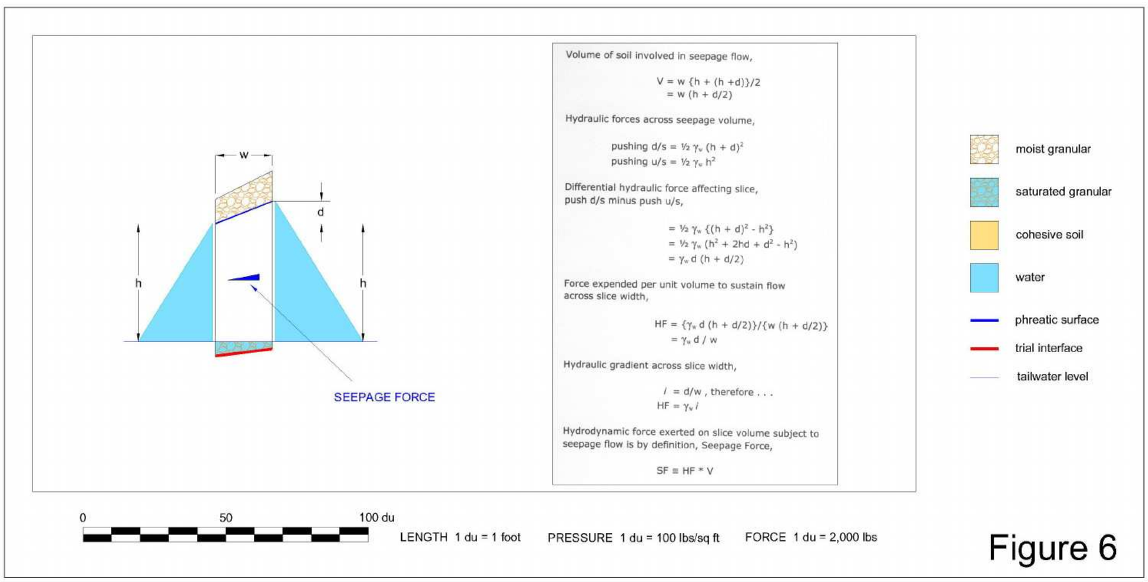


Figure 6

The hypotenuse of the blue triangle is centred on the point of action and is drawn to scale. The orthogonal sides represent the horizontal and vertical components of SF. So we may now return to the diagrams on the right side of Figures 2 and 3 and appreciate what they portray. The attached spreadsheet gives the two components calculated for the geometry in Figure 1 as SF_H and SF_V for each of the 14 slices. The cumulative magnitudes of these force resultants are shown graphically to scale on Figures 7, 8 and 9 which depict the three phreatic surfaces which have been analysed here; Figure 7 is blank in this regard because there is no gradient to the phreatic surface and consequently there is no seepage flow. Table 1 lists the numerical values of these SF force components.

One must surely wonder why, given the prevalence and power of SF in earthdams and other embankments subject to seepage flows that their presence has apparently not been the subject of serious concern. As long ago as 1948 Donald W Taylor of MIT introduced the concept of SF, see Ref 3. More recently

(50 years later) the idea was appealed to in 1998, see Ref 4. But it never seems to have gained traction in the world of geotechnical engineering.

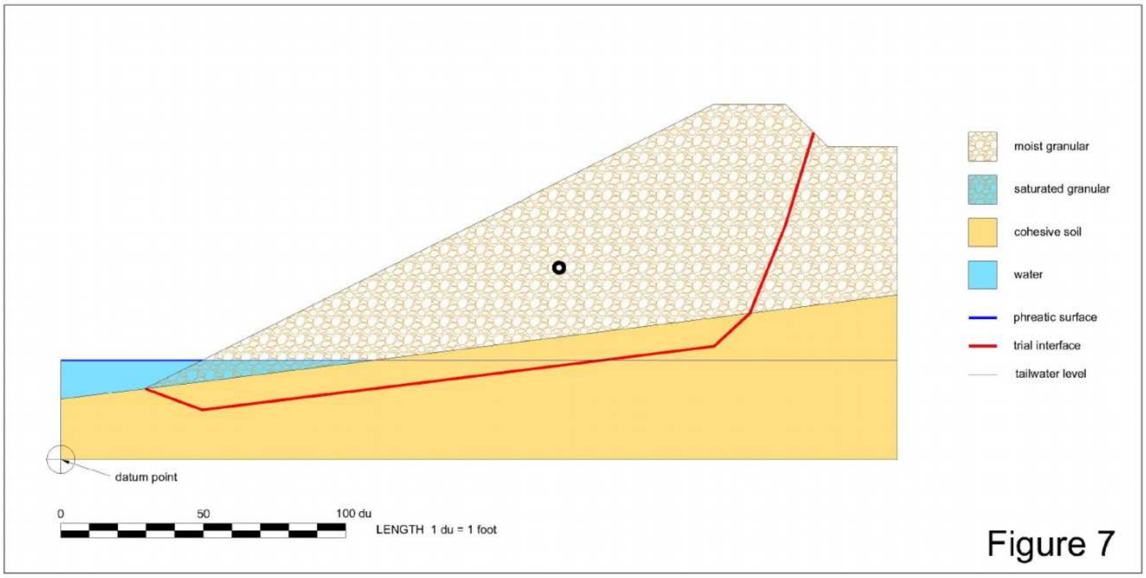


Figure 7

I can only imagine that this oversight was because of some subliminal association between flow velocity and SF, simply because D'Arcy's Law for rate of fluid flow through a porous medium ($v = i\ \ k$) and Taylor's equation for SF both contain hydraulic gradient (i) as the independent variable. It would follow (mistakenly) that since seepage velocity is always very slow, then SF must also be low. The fact that the velocity head, $h_v = \sqrt{(v^2/2g)}$ * is so miniscule that it does not register on piezometers, would only serve to add further to this misconception. But, the reality is that the magnitude of SF is not dependent in any way on soil permeability.

* For example h_v is 1.5 10^{-6} ft for permeability k=2 10^{-2} ft/sec and i =0.5

FACTORS OF SAFETY

Definition

As is appropriate for the non-circular trial interface being considered here I have chosen to define Factor of Safety [FoS] in terms of the ratio of horizontal forces: These being the ratio of shear force resisting horizontal movement of the mass above the interface, to those soil-structure and seepage forces tending to cause such a movement.

In this formulation the numerator involves the standard equation for shear strength $s = c + (\sigma_n - u) \tan\varphi$, where the dimensional variables are expressed as stress/pressure. And to convert these variables to forces, which is the policy herein, only requires that they be multiplied by the base area of the slice which is the slice width (Δw) divided by the cosine of the base inclination β. Thus, the cohesive force resisting motion is $C = c / \cos\beta$. But since the horizontal component of C, (C_H) is C times $\cos\beta$, we find that C_H is simply c times the slice width, that is, $c \cdot \Delta w$. There are two forces which contribute to the frictional resistance, these being: The effective weight of the soil column, and then, the vertical component of the Seepage Force.

Similarly, the denominator is comprised of the slice forces acting to promote lateral displacement. These are the horizontal components of the Seepage Force (SF_H) and that of the normal force (N). In addition to these slice forces, in some cases, there can be a simple single hydrostatic influence (L) introduced into the ratio-balance by standing water at either end of the section, such as the pond or a crack on the upstream side, &/or tailwater on the downstream side.

The Factor of Safety by this method is therefore =

$$\frac{\Sigma[\ C_H + (W' + SF_V) \tan\varphi \cos^2\beta\ ]}{L + \Sigma\,(SF_H + H)}$$

where, for each slice:

C_H	cohesive resistance, horizontal component
W'	effective weight of soil column
SF_V	Seepage Force, vertical component
φ	angle of frictional resistance
β	angular inclination of base
L	differential hydrostatic end forces, d/s minus u/s
SF_H	Seepage Force, horizontal component
H	horizontal component of normal force (N)

Explanation

What requires some further explanation, because of its novelty, is why and how SF should, and could, become relevant and attached to the soil-structure phase.

The phreatic surface is the locus of the level to which water would rise in a standpipe installed in the embankment. That piezometric elevation is the measure of the total hydraulic energy in the water beside the porous tip. This is called the hydraulic head above the monitored point and is the sum of the potential/pressure energy and the kinetic/velocity energy in that water. Its magnitude is usually quoted as a length (feet or metres).

If the phreatic surface is flat across the area of interest, as in Figure 7, then there is no differential in hydraulic energy within that field. Consequently, there can be no flow, because moving water needs energy to support the effort of working its way through the warren that is the soil-structure. This condition we call hydrostatic.

In a porous medium such as soil, if there is an inclination in the phreatic surface, water will flow in the direction of the maximum downward gradient (flow line). The rate of flow will be in accordance with D'Arcy's Law, where *k* (permeability or hydraulic conductivity) is the resistance to flow, a measure of the energy requirements to sustain it.

If we now switch our perspective from the water to the soil-structure, that is, from energy expended to work done, we are lead to ask the question: What is the work done on? And this is where our sister discipline, Fluid Mechanics, gives us the answer. The work is done in overcoming the hydrodynamic Drag Force [D_F] which resists the relative motion of water past the solids. D_F is proportional to the square of the relative velocity, and is composed of frictional and viscous elements. A detailed quantification of this exchange of energy is given in Ref 5.

So, in practical terms what happens during steady state seepage through a porous media as that the water, in seeking tailwater level, tends to drag the soil-structure in the same direction. In doing so it transfers by frictional and viscous drag forces' energy to the soil-structure, and this, as an intergranular compression parallel to the flow lines. This give-and-take between the liquid phase (seepage water) and the solid phase (soil-structure), from the point of view of SA, has two effects:

c. The horizontal component (SF_H) is a destabilizing influence.

d. The vertical component (SF_V), by adding to the effective normal force on the trial interface, increase shear resistance in granular soil, but brings no similar benefit to soil sections where the base soils are cohesive.

<u>Results</u>

The soil geometry depicted in Figure 1 has been analysed for horizontal stability using the hydrodynamic method (h-method) advocated here. Three positions of the phreatic surface were evaluated both for the situations where the trial interface passes mainly through the cohesive soil (c-φ), and also where the soil is made to be entirely frictional (all-φ). The results of these six stability analyses are listed in Table 1.

The first pairing, labelled "no pond", is for a condition where the phreatic surface is made coincident with tailwater level, as shown on Figure 7. This extreme position is most unlikely to be encountered in the field but is of interest here in that it abstracts any influence of SFs, thus providing a baseline situation against which other phreatic surfaces may be compared. This particular case was also used to determine what value of cohesive strength would be the equivalent of granular friction (φ = 30°), for this geometry and no SFs. It turned out to be 2,800 lb/ft^2, a value which was subsequently used in all the other analyses reported here. For this baseline condition the computed FoS is 3.0 in both cases.

TABLE 1 – Numerical Results of Stability Analyses

<table>
<tr><th colspan="2" rowspan="2">CONDITIONS</th><th colspan="2">Seepage Force components</th><th colspan="2">Factor of Safety</th><th rowspan="2">JANBU wrt h-method

% ratio</th></tr>
<tr><th>Horizontal</th><th>Vertical</th><th>h-method</th><th>JANBU*</th></tr>
<tr><td rowspan="2">no pond</td><td>c - φ</td><td rowspan="2">0</td><td rowspan="2">0</td><td>2.98</td><td>2.80</td><td>-6.0 %</td></tr>
<tr><td>all - φ</td><td>2.97</td><td>2.87</td><td>-3.3 %</td></tr>
<tr><td rowspan="2">no drain</td><td>c - φ</td><td rowspan="2">199 kips</td><td rowspan="2">41 kips</td><td>1.77</td><td>2.44</td><td>37.6 %</td></tr>
<tr><td>all - φ</td><td>1.32</td><td>1.79</td><td>35.5 %</td></tr>
<tr><td rowspan="2">chimney drain</td><td>c - φ</td><td rowspan="2">152 kips</td><td rowspan="2">102 kips</td><td>1.86</td><td>2.60</td><td>39.8 %</td></tr>
<tr><td>all - φ</td><td>1.90</td><td>2.52</td><td>32.5 %</td></tr>
</table>

*   Using Janbu's equation as shown in Ref 6 without applying either the strain compatibility function $m\alpha$, nor the f_0 allowance for interslice forces. If these multiplication factors were used the Janbu "corrected" FoS would be increased by 15.6% for c-φ computations, and 6.8% for those of the full frictional sections, on average.

The second pairing is for the high phreatic surface shown on Figure 8, and produced the results shown on the table across from the "no drain" label, is meant to depict a natural slope of embankment where there has been no effort to control the seepage artificially. It needs to be declared that this surface was not determined from any combination of permeability components, but rather, it was chosen simply for illustrative purposes. Here we see that there is a noteworthy difference between the FoS calculated for the two soil types. And in

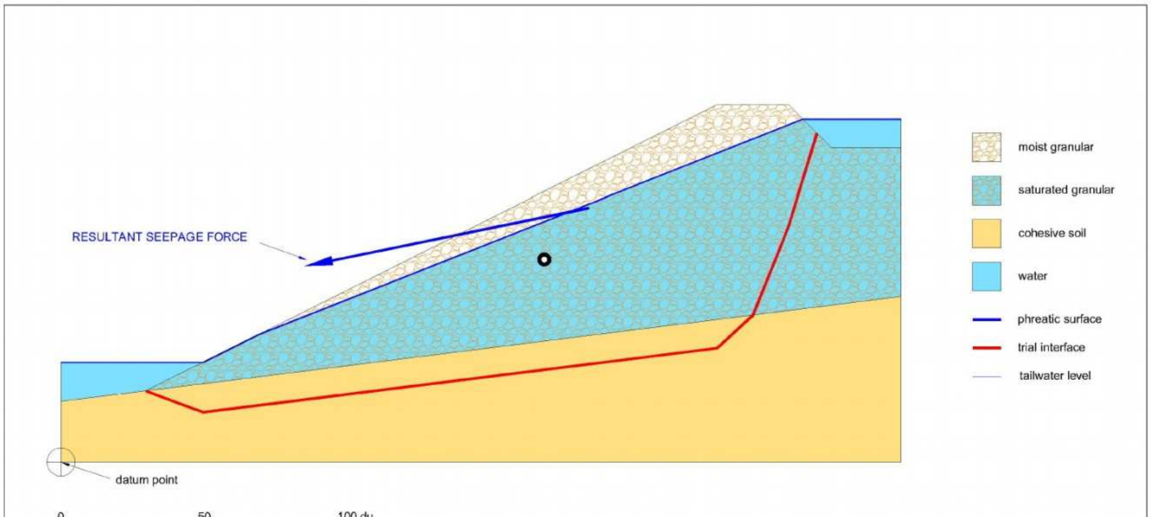


Figure 8

comparison with the "no pond" condition the FoS for the c&φ embankment fill drops by 41% while the reduction in stability for the fully frictional material is far greater, at 56%.

The third pairing is for the low, artificially depressed, phreatic surface shown in Figure 9. The FoS results are listed in the table across from the "chimney drain" label. Both result are virtually the same at about 1.9.

<u>Discussion of Results</u>

Let's first look at the numbers on Table 1 which were computed by the stability analysis method advocated herein, and listed under the column heading "h-method".

With reference to Figure 7 we see that the "no pond" pair of results both give FoS = 3.0, which should come as no surprise because, as stated above, the strength parameters were manipulated to bring about this equivalence. Both SF components are zero, again a result of deciding to set the phreatic surface at tailwater level.

In the case of the second pairing, for the situation depicted in Figure 8 and with results listed against the label "no drain", the big difference between the c-φ and the all-φ needs comment. And this can be explained as follows: the high phreatic surface means that most of the overburden is buoyant and while this has little impact on the resistance of the c&φ section, it results in a large strength reduction in the case of the all-φ section because W' is much reduced.

The third pairing, for the physical situation depicted in Figure 9, the results are listed against "chimney drain" in Table 1. The fact that the c-φ result for the

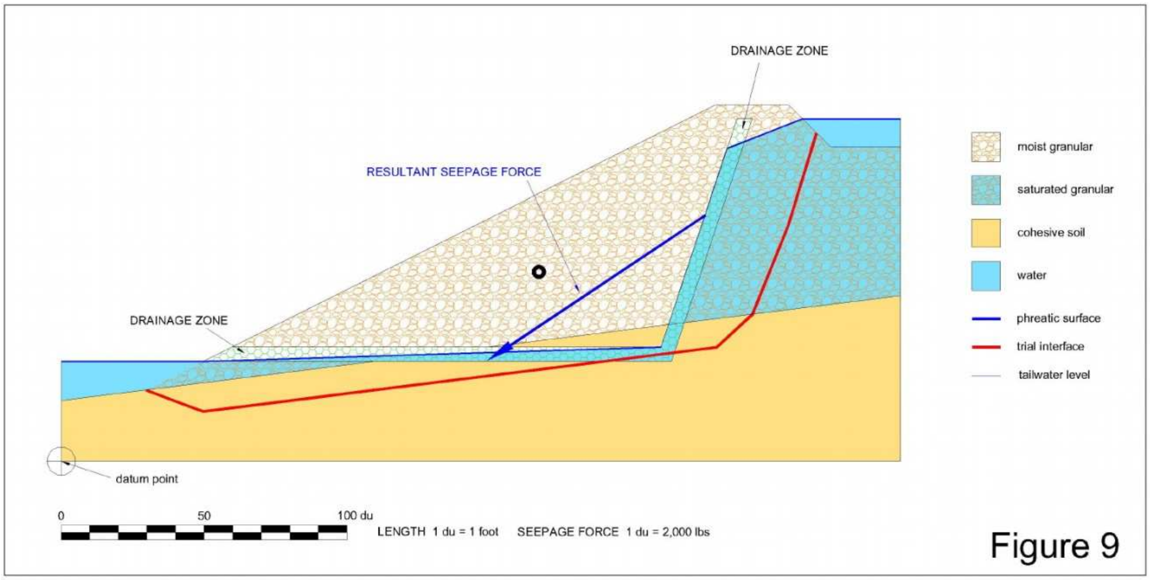


Figure 9

"chimney drain" didn't show much improvement over the equivalent value for the "no drain" situation came as a bit of an unpleasant surprise for me since I favour this design approach. Nevertheless, I suppose that I can still take some consolation in knowing that this type of drain removes concerns about differential horizontal-to-vertical shell permeability which could result in seepage breakout on the downstream slope face. The reason for this disappointing finding is twofold: the cohesive resistance does not benefit from SF_V while still having to provide resistance against its complementary SF_H destabilising influence; and, in the short stretch where the base is frictional the trial interface is quite steep and consequently cos β is a small number.

It is now necessary to compare these h-method results with those computed by a recognized and authoritative stability analysis program to see where we stand. The Janbu solution for the determination of the stability of a sliding mass on a non-circular slip surfaces is the obvious choice. Both approaches use the method of slices and both assess/determine stability on the basis of horizontal equilibrium.

For the purpose of making the clearest comparison between the two methods a liberty has been taken with Janbu, and this explains the "*" attached to those results wherever they appear here. Neither of the factors $m\alpha$ nor f_o have been applied to the basic FoS calculated. Since these factors are equally applicable to the h-method, leaving these sophistications out is purely to eliminate redundancies in order to provide clarity; it does not in any way degrade the validity of the comparison. In any event, applying them would simply entail multiplying the results of both the Janbu and h-method numbers by (in the tabulated order), 1.16, 1.15, 1.15 for the c-φ results, and 1.08, 1.05, 1.07 for all-φ results. In their own way these lists highlight the rather arbitrary nature of such a refinement.

The final step taken for the sake of making for the least complicated comparison between these two stability analysis procedures was to make the unit weights of the cohesive and the frictional soil the same. This was accomplished by setting both void ratios at 0.5 for both soil types. Since now we are dealing with the same embankment sections, phreatic surfaces, soil strength parameters, and unit weights, we can let the computed FoS from each procedure speak for themselves.

Now turning to the FoS values listed under the column heading "JANBU*".

We see that the values for the benchmark "no pond" condition are 2.8 for the c-φ soils and 2.9 for the all-φ case. The column immediately to the right labelled "JANBU wrt h-method % ratio" shows that the h-method produces on average results which are 4.6% higher than JANBU. This, I think, is a reasonably good level of agreement.

When it comes to the "no drain" pair things are quite different. The JANBU c&φ value is higher than the h-method by 38% and the all-φ FoS exceeds this method's number by 36%. It is worth noting that apart from this wide gap, the value given for the frictional case is again, as in the h-method calculation, much less than its cohesive counterpart. And here too it is for a similar, but somewhat different reason. Here the shear strength is depressed because of high pore water pressures.

The comparisons for the "chimney drain" set shows the same inclination as that of the "no drain". The Janbu pair of values exceed those of the h-method by 40% and 33%.

The above comparisons show that treating the seepage water within a slope as a matter of hydrodynamics, which it truly is, yields FoS values far lower than those which are arrived at when seepage is dealt with as a hydrostatic condition. In consequence, it must be concluded that our standard formulations of stability analyses which deal in pore water pressures, rather than seepage forces, err significantly on the unsafe side.

UTILITY IN PRACTICE

Once the existence of Seepage Forces are acknowledged, and are given their rightful place in stability analysis, it will become possible to more rationally address concerns pertaining to the three classical states of earthdam stability. These states are referred to as the "end of construction", "steady state seepage" of the downstream slope, and the effects of "rapid drawdown" of the pond on the upstream face. Briefly,

e. End of Construction, that is, before the reservoir is impounded, is very much a matter of zoning and material types. And since there is no overt seepage one would be inclined to dismiss the h–method as having no application. Nonetheless it may prove to be the best way to do a post-mortem on the failure of Fort Peck dam, which occurred even before construction was complete.

f. Stability of the downstream slope at both Tarbela and Bennett dams under the presumed conditions of Steady State Seepage caused unwarranted concerns. This is because, as argued in Ref 7, this condition did not come about for several decades after the ponds were filled, and then its natural manifestations resulted in surprise. This h-method of SA, in combination with what is known from the physics of 3-phased materials/systems (see Ref 7, Section 9) could have alleviated much ado, especially at Bennett.

g. In the case of Rapid Drawdown of the reservoir the reality of Drag forces become apparent when/where one can easily visualize the zone of saturation lagging behind the receding pond level. What may not be as clear is the fact that when the reservoir is next impounded, the saturation of the upstream shell may not happen quickly, and perhaps never. Again, I suggest Ref 7 as the source of numerical data on this transient condition.

Initially, I hoped that this h-method could be extended to deal with the dynamics involved in debris flows and rockfalls, but it soon became obvious that that ambition would be at odds with some of the basic principles on which this particular approach is predicated: While these two phenomena have some things in common with embankment slopes approaching incipient failure, nevertheless, they differ in fundamental ways that preclude their treatment by means of the h-method presented here. This is because both types of failure are non-steady states, and are based on the total collapse of a liquefied mass moving rapidly. These two also involve volume changes with time as the hyper-pressurized fluid (water/air, respectively), vent from the composite mass, eventually causing it to come to rest once these excesses of fluid have escaped the moving mass. So, this venture must be left for another time.

CONCLUDING REMARKS

It is a mistake to treat saturated soil as a fixed entity. It is not. Saturated soil is a two-phase material where the two phases are not locked together, they do not act in concert. The water is entirely free to move within the soil-structure, but can only avail itself of this freedom of relocation when/while it has the energy to do so. This is because water must expend energy to do the work entailed in overcoming the drag resistance the soil-structure puts in the way of such relative motion between the phases.

Hydraulic head, as depicted by the phreatic surface, is a measure of the available energy in the water. Hydrodynamic energy is available to the liquid phase whenever/wherever it finds itself under the dominion of a hydraulic gradient. Where there is no hydraulic gradient the situation is hydrostatic: The water can't move. Therefore, other than by buoyant effect, the pore water has no power to affect the soil-structure. Seepage Force is a convenient way to quantify the particular work-energy equation in governance here.

Current SA formulations ignore this dynamic when they simply equate total weight to effective weight plus the pore water force, that is, when in terms of pressure, the effective overburden pressure is set equal to the total overburden pressure minus the hydrostatic pore water pressure, as in the shear strength equation $s = c + (\sigma_n - u) \tan \varphi$.

There is a demonstrable need, as shown here, to upgrade the way in which the geotechnical community currently assesses the stability of natural slopes and earthfills subject to seepage flow, such as embankment dams or tailings dams. Personally I view this situation as being quite serious. But nevertheless, these days, geotechnical departments are gradually retiring into minor adjuncts of Environmental engineering departments. This retreat is premature – much too soon - our work is far from done !

END OF ESSAY

Written by: William E Hodge
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